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Numerical estimation of the initial hinge-line irregularity required for the development of sheath folds: A pure shear model

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ABSTRACT

Non-cylindrical sheath folds often develop by the accentuation of inherent geometrical irregularities along the hinge line during progressive deformation. This paper presents an analysis of the growth of such non-cylindrical folds in pure shear as a function of a non-dimensional initial irregularity factor ζ_o ($\zeta_o = -\log_{10}[A_o/W_o]$, where A_o and W_o are the amplitude and the wavelength of a sinuous hinge line, respectively). Curvature accentuation of the hinge line produces sheath-like non-cylindrical folds in passive layers only when $\zeta_o \leq 1$. Folds with $\zeta_o > 1$ remain weakly or moderately non-cylindrical even after a large finite strain ($\lambda = 36$). An increasing flattening strain ($1 > k$ value > 0) component in the deformation reduces the degree of non-cylindricity. We extend the analysis into layered systems with viscosity contrasts (i.e. viscosity ratio, R), considering a strain-partitioning factor λ_f (the strain ratio between the fold hinge and bulk medium). Using finite element methods, it is shown that λ_f can be small even for low values of R (4–8). Consequently, strong non-cylindricity does not grow in higher-viscosity layers for $\zeta_o = 1$. We ran experiments using three-dimensional finite element models to investigate the additional effects of mechanical bending on curvature accentuation of the hinge line. These experiments also show little accentuation of hinge curvature irrespective of R, when $\zeta_o > 1$. However, for $\zeta_o \leq 1$, hinge-line curvature accentuates moderately when $R < 8$. It follows from our analysis that strongly non-cylindrical folds cannot develop unless the magnitude of initial hinge-line irregularity is sufficiently large (i.e. $\zeta_o \ll 1$). We also discuss the implications of the numerical results for pure shear models in the analysis of sheath folds in shear zones.

1. Introduction

The term *sheath fold* has been coined to describe non-cylindrical folds with strongly curvilinear hinge lines and gentle curvi-planar axial surfaces (Carreras et al., 1977; Quinquis et al., 1978; Minnigh, 1979; Ramsay, 1980; Alsop and Holdsworth, 2004, 2007). These are characterized by hinge curvature angles (δ) greater than 90° (Fig. 1a, Alsop and Holdsworth, 2006; Alsop and Carreras, 2007). In extreme cases, δ may be nearly 180° and the fold geometry resembles that of a flattened test tube (Turner and Weiss, 1963; Tobisch, 1966; Carreras et al., 1977; Williams and Chapman, 1979; Skjernaa, 1989; Ez, 2000; Alsop and Carreras, 2007). Sheath folds occur on varying scales (Alsop et al., 2007). Some workers have documented "mega-scale sheath" structures from field studies (Hibbard and Karig, 1987; Goscombe, 1991; Alsop and Holdsworth, 1999; Searle and Alsop, 2007). A variety of kinematic models have been used to explain the development of sheath folds. Ramsay (1980) presented a model that takes into account passive accentuation of primary geometrical irregularities on layers to explain the development of strongly non-cylindrical folds in simple shear. According to this model, incipient initial non-cylindrical curvatures accentuate progressively, and the folds tend to be strongly non-cylindrical with increasing strain (Fig. 1b; Quinquis et al., 1978; Ramsay, 1980; Cobbold and Quinquis, 1980; Kuiper et al., 2007). During these geometrical modifications, the hinge lines rotate in the axial plane, and tend to align themselves parallel to the extension direction (Sanderson, 1973; Escher and Watterson, 1974; Bell, 1978; Jiang and Williams, 1999). Recent studies show that the type of curvature accentuation depends greatly on the nature of deformation, i.e. simple shear vs, general shear vs constrictional shear, and that each type of deformation produces characteristic three-dimensional shapes of sheath structures (Alsop and Holdsworth, 2006).

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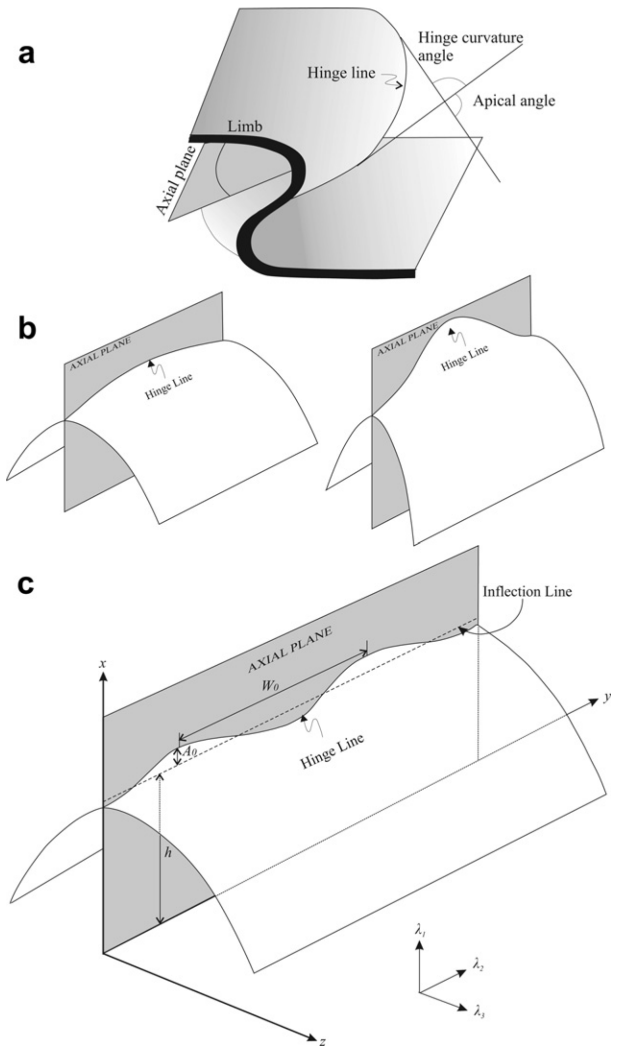

Fig. 1. (a) A three-dimensional view of schematic sheath folds, showing geometrical parameters: apical angle, hinge curvature angle and hinge line (after Alsop and Carreras, 2007) (b) A cartoon illustrating the development of sheath geometries by curvature accentuation of the hinge lines in pure shear. Note that the hinge line rotates towards the extension direction (λ_1) in the fold's axial plane. (c) Consideration of geometrical irregularities along hinge lines in the form of a sine wave with wavelength W_o and amplitude A_o. A Cartesian reference frame (*xyz*) is chosen with *xy* located along the axial plane, and the *y*-axis parallel to the inflection line of the sinuous hinge line. *h* is an arbitrary height of the inflection line from the *xz* plane. The kinematic reference frame (λ_1, λ_2, λ_3) correspond to the structures shown in (b) and (c).

Several workers have shown that sheath folds can nucleate by the process of buckling in mechanical layers or foliations in shear zones (Ghosh and Sengupta, 1984; Ez, 2000; Alsop and Holdsworth, 2004; Mandal et al., 2004; Carreras et al., 2005). Buckle folds always grow with an initial non-cylindricity, the curvature of which can accentuate due to accumulation of homogeneous strain in the folded layers (Ghosh and Sengupta, 1984). Recent experimental studies, however, demonstrate that non-cylindricity is unlikely to increase significantly if the viscosity ratio is larger than 10 (Marques et al., 2008). According to these experimental findings, sheath folds of buckling origin are possible for low-viscosity contrasts, allowing considerable layer-parallel strains. By contrast, the classical theory of buckling instability for viscous layers indicates that folds cannot grow in high-viscosity layers unless the viscosity ratio is large (Biot, 1965; Ramberg, 1964; Hudleston, 1973). We therefore need to refine our understanding how post-buckling fold shape modifications can involve large ductile strains. Furthermore, the geometry of non-cylindrical folds formed by buckling instabilities can be modified into diverse shapes, such as bull's-eye-fold, analogous-eye-fold and cat's-eye-fold, depending upon the nature of the bulk deformation (Alsop and Holdsworth, 2006).

Several workers have shown that sheath folds in shear zones can also grow by other mechanisms. For example, initial hinge geometries can have their curvatures passively accentuated by the heterogeneous strain fields that develop around small-scale objects, such as rigid inclusions, giving rise to sheath-like structures (Marques and Cobbold, 1995; Rosas et al., 2002). On the other hand, Mies (1993) has shown passive amplification of initial structures in plastic non-coaxial deformations.

Most of the genetic models, based on either buckling or passive folding take into account initial non-cylindricity with curvilinear hinges (e.g. Ramsay, 1980; Ghosh and Sengupta, 1984; Alsop and Holdsworth, 2002). We, therefore, need to quantify the magnitude of initial hinge irregularities required for the development of sheath folds over the possible range of geological strains. Using simulation experiments, this study aims to present such an analysis. In these experiments we consider that initial non-cylindrical shapes accentuate due to ductile strains in passive or mechanical layers under pure shear deformation conditions. Ez (2000) has shown the shape modification of non-cylindrical curvatures with an initial dome and basin like geometry. However, both field and experimental observations reveal that incipient folds nucleate with non-cylindrical curvatures containing wavy hinge lines (cf. Ghosh and Ramberg, 1968; Dubey and Cobbold, 1977; Ramsay and Huber, 1987). This type of non-cylindrical curvature can also accentuate with increasing ductile strain in the layer, and finally attain sheath-like geometry (Ramsay, 1980; Ghosh and Sengupta, 1984). In this study we consider a non-dimensional irregularity factor ζ_o on a logarithmic scale as $-\log_{10}(A_o/W_o)$, where A_o and W_o are the initial amplitude and wavelength of the hinge-line irregularity, to describe the initial geometry of hinge lines. This factor is a measure of the initial non-cylindricity of folds, which may be of different origins. We performed a set of buckling experiments to test the magnitudes of ζ_o possible for folds produced by buckling in pure shear. A 1 mm thick sheet of modeling clay was embedded in putty, and then deformed by layer-parallel shortening. The deformed model produced a large number of discrete, elongate folds with non-cylindricity (Fig. 2). The amplitude and length of hinges were measured from the layer surface. Their frequency diagram shows that the magnitude of geometrical irregularities is in the order of 10^{-2}, i.e. $\zeta_o = 2$ (Fig. 3). We shall see later that buckle folds with hinge irregularities of this order are unlikely to produce sheath geometries during pure shear. Initial non-cylindrical deflections resulting from primary processes can also produce sheath structures. For example, penecontemporaneous structures, such as convoluted bedding and slump folds can accentuate their initial curvatures without the development of high finite strains (e.g. Roberts, 1989; Alsop and Holdsworth, 2004). There are also well-documented examples of highly curvilinear folds formed under very modest finite strains in some upper crustal transpression zones (e.g. Holdsworth and Pinheiro, 2000; Holdsworth et al., 2002). The magnitudes of initial non-cylindricity of these folds may be large.

In this paper we present an analysis of sheath development as a function of the initial hinge-line irregularity factor ζ_o, considering pure shear deformations. Our estimates show that an irregularity is likely to grow in passive layers, and produce sheath-like non-cylindricity in plausible finite strains only when ζ_o reaches a threshold value. Using finite element methods, we also determine the ζ_o values necessary for the development of strongly non-cylindrical folds in high-viscosity layers embedded in a soft viscous

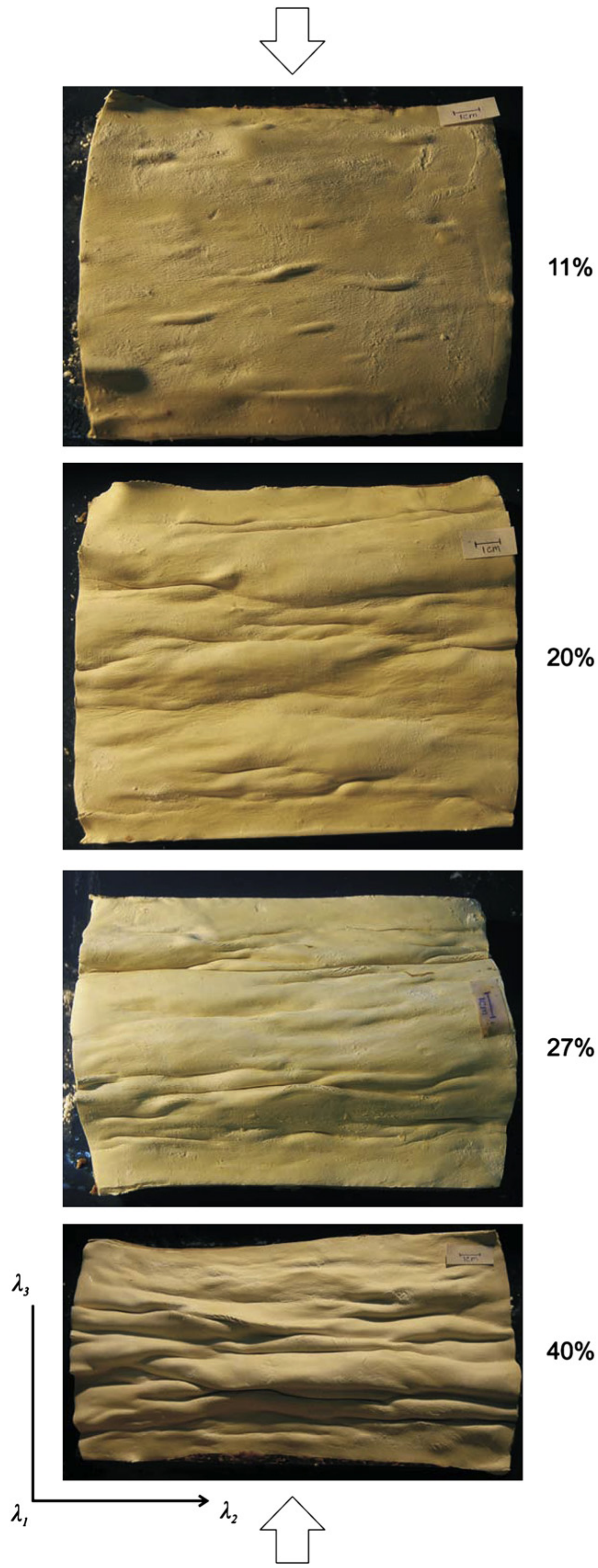

Fig. 2. Buckling experiments on a stiff layer (plasticine) embedded in putty blocks. These physical models illustrate the nucleation and growth of discrete buckle folds with curved hinges. The upper putty blocks have been removed to reveal the 3D geometry of the discrete folds in the stiff layers.

medium. This analysis takes into account a strain-partitioning factor (the ratio of finite strains in the fold hinge and in the bulk medium) as a function of the viscosity ratio (R). We show that it is difficult for sheath-like non-cylindricity to develop in a high-viscosity layer ($R=8$) even for large magnitudes of initial hinge-line irregularity. We present a series of experimental results obtained from 3D finite element models, which were employed to investigate if there is any additional effect from active bending of the hinge zone in curvature accentuation. The results suggest that folds remain almost cylindrical unless ζ_o is small. We also discuss the results of pure shear models in the context of sheath fold structures in shear zones.

2. Passive accentuation of hinge-line curvature and fold non-cylindricity

2.1. Theoretical considerations

Consider a plane tight fold containing geometrical irregularities on its hinge line in the form of a sine wave (Fig. 1c). By choosing a Cartesian reference frame, xyz, the initial hinge-line irregularity can be expressed by

$$(x-h) = A_o \sin \frac{2\pi}{W_o} y, \tag{1}$$

where A_o and W_o are the amplitude and wavelength of the irregularity. h is the average distance of the hinge line from the yz reference plane. In this analysis we define a geometrical factor ζ_o to describe the magnitude of the initial irregularity as

$$\zeta_o = -\log_{10} \frac{A_o}{W_o} \tag{2}$$

The main objective of this exercise is to estimate the threshold ζ_o values necessary for the development of strongly non-cylindrical folds over possible geological strains.

We assume that the fold structure undergoes accentuation in response to an incremental bulk strain, which is represented by the following tensor:

$$\varepsilon_{ij} = \begin{pmatrix} \sqrt{\lambda_1}-1 & 0 & 0 \\ 0 & \sqrt{\lambda_2}-1 & 0 \\ 0 & 0 & \sqrt{\lambda_3}-1 \end{pmatrix} \tag{3}$$

where λ_1, λ_2 and λ_3 are the principal quadratic elongations (Fig. 1c).

It should be noted that this analysis assumes that the axial surface continuously tracks the principal plane of bulk strain ($\lambda_1\lambda_2$ plane). We impose a constant volume condition during deformation by taking:

$$\lambda_1\lambda_2\lambda_3 = 1 \tag{4}$$

The analysis takes into account both plane strain ($\lambda_2=1$) and flattening strain ($\lambda_2>1$) conditions. We developed a computer code in MATLAB for numerical simulations of accentuation of initial hinge-line curvature of tight planar folds in three dimensions using the following procedure. In this analysis we use some terms in describing the sheath geometry of folds, such as apical angle (ϕ) and interlimb angle (Fig. 1a; cf. Alsop and Carreras, 2007). ϕ is a geometrical parameter indicating the degree of hinge-line curvature.

The sinuous hinge (Eq. (1)) progressively accentuates its curvature, resulting in an increasing non-cylindricity of the fold structure. The maximum hinge curvature of the fold at any instant

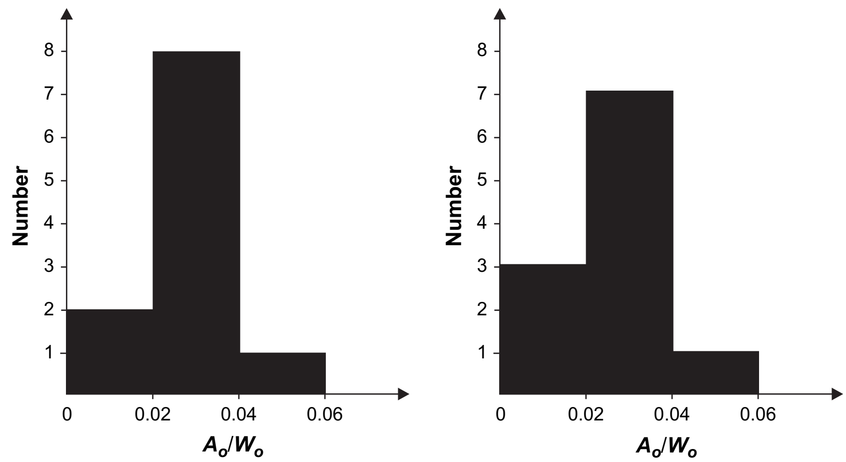

Fig. 3. Frequency diagram for the magnitude of geometrical irregularities (amplitude/wavelength ratio) along fold hinge lines obtained from buckling experiments, as illustrated in Fig. 2. Models corresponding to the frequency diagram on the right-hand side are not presented here.

can be obtained analytically in the following way. Consider the sinuous hinge line in the deformed model as:

$$x = a \sin \frac{2\pi y}{w} \tag{5}$$

where a and w are the amplitude and wavelength of the accentuated sinuous hinge. The radius of curvature (r) can be derived from the following equation:

$$r = \frac{\left|1 + \left(\frac{dx}{dy}\right)^2\right)^{\frac{3}{2}}\right|}{\left|\frac{d^2x}{dy^2}\right|} \tag{6}$$

Differentiating Eq. (5) and replacing the derivatives in Eq. (6), we have the normalized radius of curvature at $y = w/4$,

$$r^* = \frac{r}{w} = \frac{1}{4\pi^2 k'} \quad \text{where} \quad k = \frac{a}{w} \tag{7}$$

Taking the logarithm of both sides and after some algebraic manipulation of Eq. (7), we can express the normalized hinge-line curvature (ρ) as a function of the geometrical factor ζ:

$$\rho = \exp\left[\frac{\zeta + 2\log_{10} 4\pi}{\log_{10} e}\right] \tag{8}$$

Considering the tangents at the inflection points, we can determine the hinge angle using the following mathematical steps:

$$\frac{dx}{dy} = \frac{2\pi a}{w} \cos \frac{2\pi y}{w} \tag{9}$$

At $x = 0$, the tangent slope is:

$$\tan\theta = \frac{dx}{dy} = \frac{2\pi a}{w} = 2\pi k \tag{10}$$

The apical angle ϕ can be then expressed as:

$$\tan 2\phi = -\frac{2\tan\theta}{1 + \tan^2\theta} \tag{11}$$

$$\tan 2\phi = -\frac{4\pi k}{1 + 4\pi^2 k^2} \tag{12}$$

$$\tan 2\phi = -\frac{4\pi e^{(\zeta/\log_{10} e)}}{1 + 4\pi^2 e^{(2\zeta/\log_{10} e)}} \tag{13}$$

Based on the above principle, a series of simulation experiments was performed to determine the order of magnitude of the initial hinge irregularity that may lead to development of strong non-cylindricity. We ran experiments in plane ($\lambda_2 = 1$) as well as in flattening strain conditions ($\lambda_2 > 1$) with $\zeta_o = 1$, 2 and 3.

2.2. Experimental results

Experiments with $\zeta_o = 3$ do not develop any significant non-cylindricity ($\rho = 0.2$, Eq. (8)) even when the bulk finite strain is very large ($\lambda_1 = 36$, Fig. 4a), and the deformed fold profile is extremely tight (interlimb angle $< 4°$, Fig. 5a). The calculated hinge angle (Eq. (13)) is greater than 176°. With further increases in finite strain the interlimb angle would be close to zero. It thus appears that initial folds with $\zeta_o = 3$ are unlikely to develop sheath-like non-cylindricity by passive accentuation of hinge curvature under pure shear conditions.

Experiments with $\zeta_o = 2$ developed a non-cylindrical 3D fold geometry with considerable hinge curvatures ($\rho \approx 2$, $2\phi = 166°$) (Fig. 4b). However, the overall structure is unlike a typical sheath fold even after large deformations. The folds rather resemble plane non-cylindrical structures with culminations and depressions (Ramsay, 1967). It is extremely tight with an interlimb angle of 3° (Fig. 5b).

The growth of strongly non-cylindrical folds was evident in simulations with $\zeta_o = 1$ (Fig. 4c). The hinge angle became about 38° when $\lambda_1 = 36$. The three-dimensional fold geometry shows a strong curvature of the hinge line ($\rho \approx 24$, $2\phi = 68°$, Eq. (8)), which closely resembles a sheath structure. The fold is extremely tight (Fig. 5c). This set of experiments suggests that the initial hinge curvature ρ_o should be in the order of 4 for the development of strong non-cylindricity. Folds with lower initial hinge curvatures are unlikely to produce typical sheath folds under pure shear conditions.

Sheath fold shapes are controlled by the type of deformation, i.e. general shear vs, shear with flattening or constriction (Alsop and Holdsworth, 2006). We performed experiments with $\lambda_2 > 1$ parallel to the wavy hinge of early non-cylindrical folds with varying ζ_o values. For a given ζ_o, the degree of curvature accentuation weakens with increasing flattening strain components, as reported from earlier studies (e.g. Jiang and Williams, 1999). Folds with $\zeta_o = 2$ and 3 do not develop any perceptible non-cylindricity when $\lambda_2 = 4$ (Fig. 6). Experiments with $\zeta_o = 1$ produced a sheath-like geometry for $\lambda_2 = 1$, which transforms into a plane non-cylindrical fold with large hinge angles (>60°) for $\lambda_2 > 2.25$.

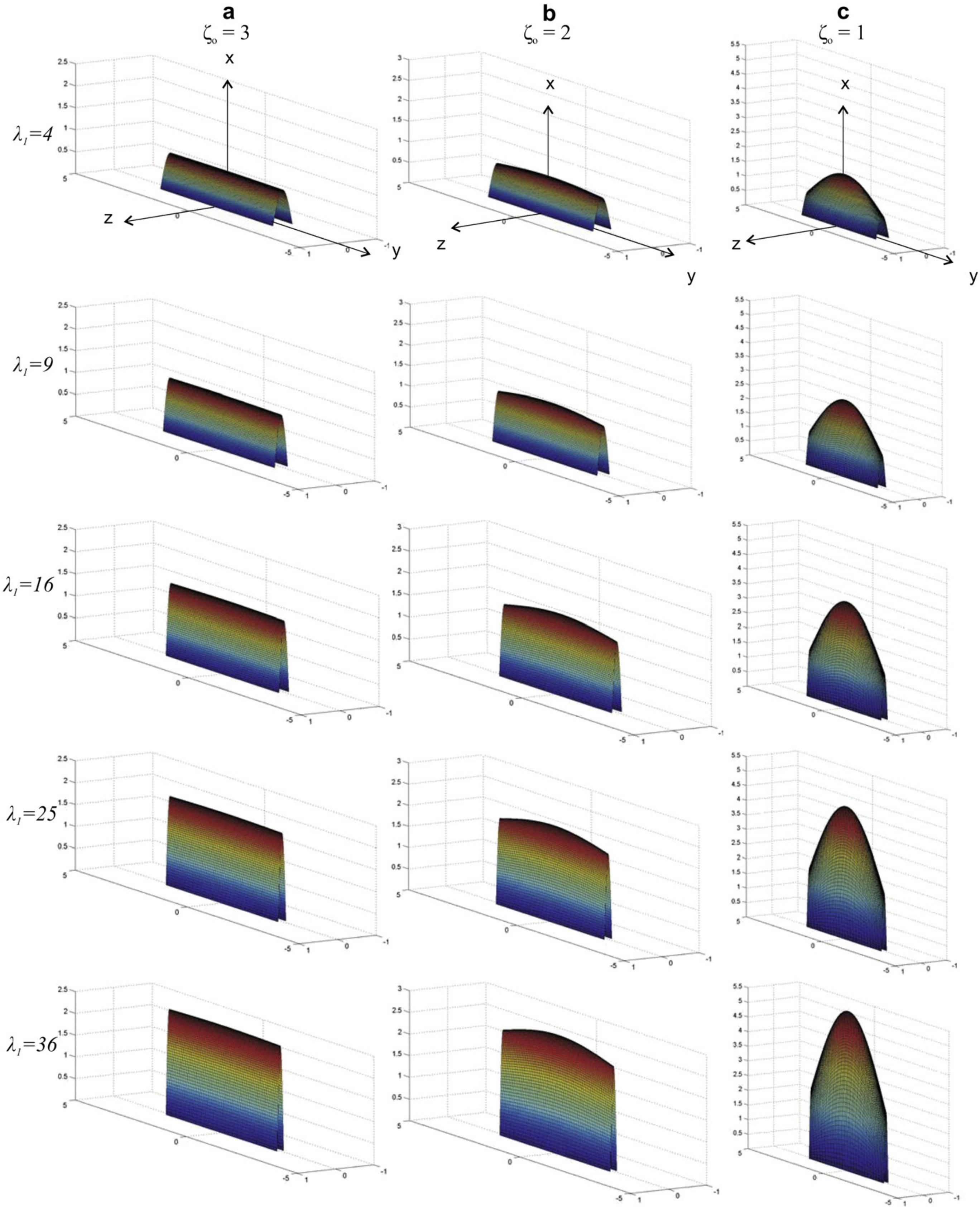

Fig. 4. Simulations of plane non-cylindrical folds in passive layers under plane strain condition ($\lambda_2 = 1$). $\zeta_o = 3$, 2 and 1 in (a), (b) and (c) respectively. $\zeta_o = -\log_{10}(A_o/W_o)$ where A_o and W_o are the initial amplitude and wavelength of geometrical irregularities along the hinge lines. λ_1: bulk quadratic elongation. Colour contours (blue to red) indicate increasing vertical elevations of points on the fold surface with respect to the reference plane. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

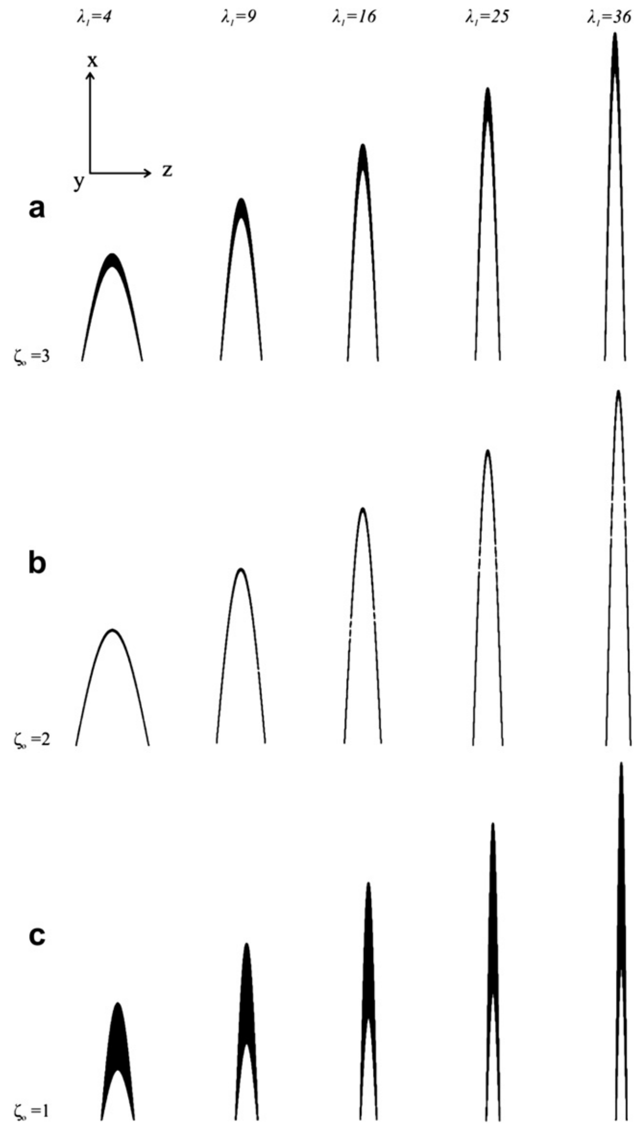

Fig. 5. View of non-cylindrical folds along the hinge line (i.e. normal to xz plane). $\zeta_o = 3$, 2 and 1 in (a), (b) and (c) respectively. Note that the inner and outer curves indicate fold profiles at the front and the middle sections of the sheath structure.

3. Non-cylindrical folding in high-viscosity layers

3.1. Method of analysis

It may be recalled that accentuation of inherent hinge geometries is determined by the ductile strain developed in the hinge zone. In case of folding in a high-viscosity layer embedded within a relatively low-viscosity medium, the strain partitioning between the fold hinge and the bulk medium would be an additional controlling factor. In this section we analyze the non-cylindricity considering a strain-partitioning factor:

$$\lambda_f = \frac{\lambda_{1h}}{\lambda_{1b}} \tag{14}$$

where λ_{1h} and λ_{1b} are the quadratic elongations at the hinge and in the bulk medium. These are obtained from the following equations:

$$\lambda_{1h} = (t_l/t_0)^2 \quad \text{and} \quad \lambda_{1b} = (l_l/l_0)^2, \tag{15}$$

t_l and t_o are the axial plane thickness at the hinge for a given bulk finite strain and the initial layer thickness respectively, and l_l and l_o are the model lengths after and before deformation.

For passive layers, $\lambda_f = 1$, whilst, $\lambda_f < 1$ for high-viscosity layers. Evidently, layers with very large competence contrasts form folds with little or no hinge strain, as observed in Class 1B folds (Ramsay, 1967). In that case, $\lambda_f \approx 0$ for large bulk strains. Using a finite element method we estimated λ_f for layers with different competence contrasts in the following way.

A 2D elastico-viscous model was chosen (cf. Passchier and Druguet, 2002), which consisted of a wavy high-viscosity layer with a shear modulus μ_L and co-efficient of viscosity η_L set within a matrix of shear modulus μ_M and viscosity η_M. Assuming a very slow rate of shortening, an approximate arc-length of buckle fold can be obtained from Biot's (1957) equation:

$$W_d = 2\pi h^3 \sqrt{\frac{\eta_L}{6\eta_M}} \tag{16}$$

The rheological parameters chosen during the finite element modeling are described in Table 1. Models were developed using the commercial finite element code of ANSYS® (version 11). The initial fold wavelength in the model was obtained from Eq. (16). The model was then deformed in a plane strain condition ($\lambda_2 = 1$) with the principal shortening (λ_3) perpendicular to the axial plane of the folds. The hinge strain (λ_{1h}) was estimated from the deformed fold shape (Fig. 7), and plotted as a function of the bulk strain (λ_{1b}). The plots show that λ_{1h} increases more or less linearly with λ_{1b} (Fig. 8). The slope of $\lambda_{1h} - \lambda_{1b}$ variations gives an estimate of the strain-partitioning factor λ_f, which is inversely proportional to the viscosity ratio $R = \eta_L/\eta_M$.

3.2. Simulation experiments

Using the strain-partitioning factor λ_f, we simulated the three-dimensional fold geometry as described in Section 2.1. The simulation assumes that λ_f remains uniform along the fold hinge line. The experiments show that the accentuation of the initial hinge curvature is much less compared to that for passive layers, even when the geometrical factor $\zeta_o = 1$. We determined non-cylindrical fold geometries for viscosity ratios $R = 8$, 10 and 20 (Fig. 9). The hinge line accentuates its curvature to a negligible extent when $R = 20$ (Fig. 9a). The result is consistent with that of Marques et al. (2008), who have shown from physical experiments that sheath-like non-cylindrical folds do not develop in high-viscosity layers with $R > 10$. In our simulation experiments there is a slight increase in hinge curvature with decreasing R (Fig. 9b). However, the degree of accentuation of hinge curvature was not large even for $R < 10$. At $R = 8$, the hinge angle was about 120° for a finite strain with $\lambda_1 = 4$ (Fig. 9c). We also estimated the maximum hinge curvature, which is about 0.9 for a viscosity ratio of 20, and increases to 1.02 when $R = 8$.

3.3. 3D finite element modeling

We investigated the problem of curvature accentuation using the same finite element methods in three dimensions. The purpose of this 3D modeling was to enumerate possible degrees of accentuation of hinge curvatures resulting from a combined effect of active bending of the hinge line and its re-orientation due to homogeneous strain in layers, as described in the previous section. A segment of the fold structure may have a tendency to rotate like rigid linear elements, and orient itself parallel to the bulk stretching direction. Such a kinematic response can lead to active bending in the fold structure. Considering this mechanism, we ran experiments on

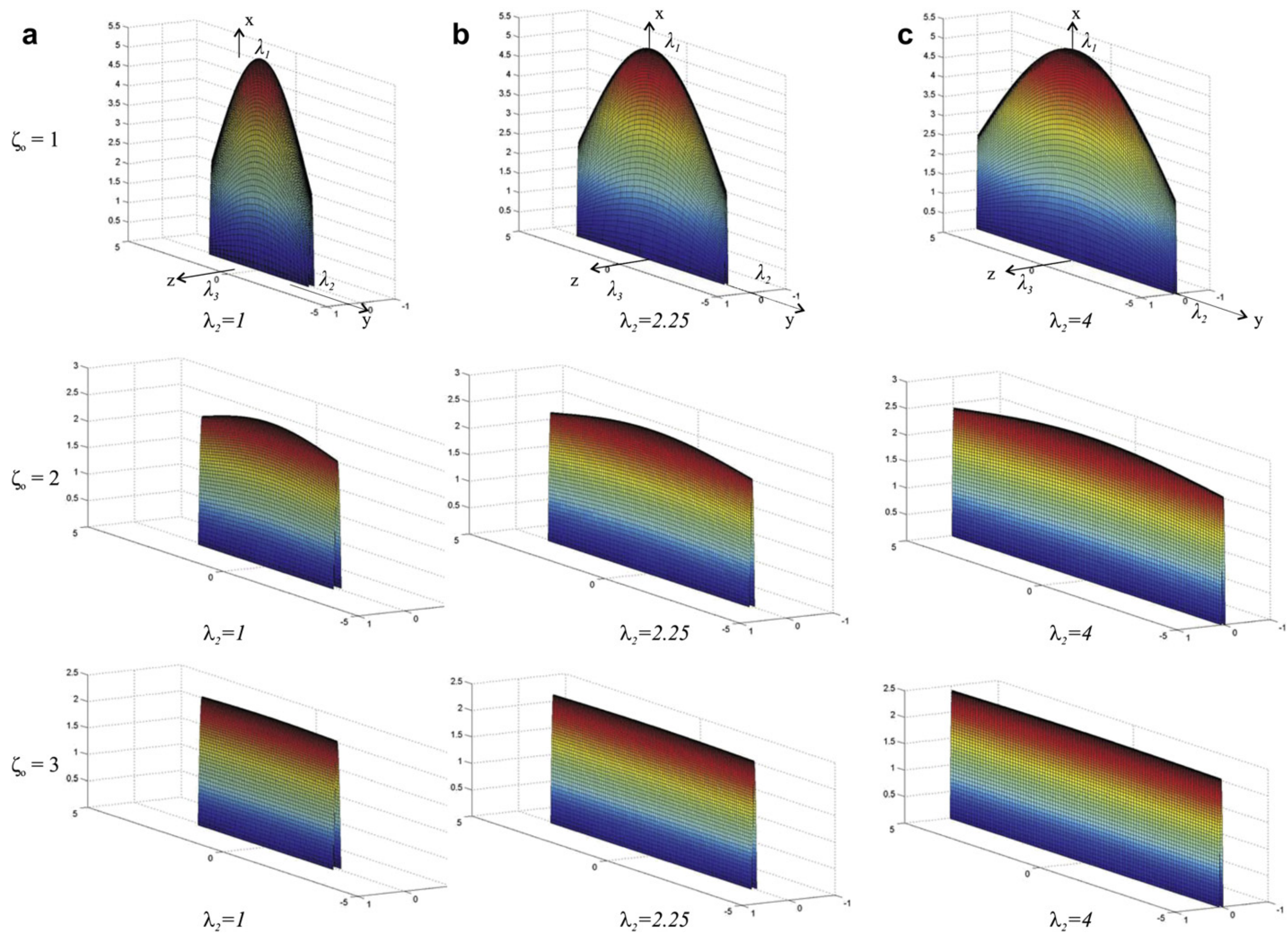

Fig. 6. Effects of flattening strain ($\lambda_2 > 1$) on the development of non-cylindrical geometries. Note the decreasing non-cylindricity with increasing flattening strain.

finite element models simulating a three-dimensional fold structure with initial geometrical irregularities along the hinge line. The rheological parameters and the fold waves were chosen as described in the previous section. Different boundary conditions and the bulk strain axes are shown in Fig. 10a.

A set of simulation experiments was performed by varying the viscosity ratio R, keeping the irregularity factor $\zeta_o = 3$. Models with $R = 8$ show that the folds remain almost cylindrical (hinge angle 174° and curvature 0.3) even when they progressively tighten and become nearly isoclinal after a bulk shortening of 50% (Fig. 10b). Accentuation of the hinge-line curvature was somewhat perceptible in models with $R = 4$. However, its magnitude was extremely low, giving rise to virtually cylindrical fold structures (Fig. 10c). The hinge angle was greater than 170°, and the maximum curvature of hinge line did not exceed 0.5. These experiments also indicate that folds in high-viscosity layers are unlikely to produce sheath-like structures, irrespective of R values, if $\zeta_o = 3$. We ran another set of experiments

Table 1
Values of rheological parameters considered in finite element modeling.

	Material				
	Viscosity	Shear mod.	Bulk mod.	Density	Runtime
Embedding medium (incompetent unit)	1.25e10 Pa-S	1.2e10 Pa	2e10 Pa	2500 kg/m^3	1 My
Layer (competent unit)	1e11 Pa-S	9.6e10 Pa	16e10 Pa	2500 kg/m^3	

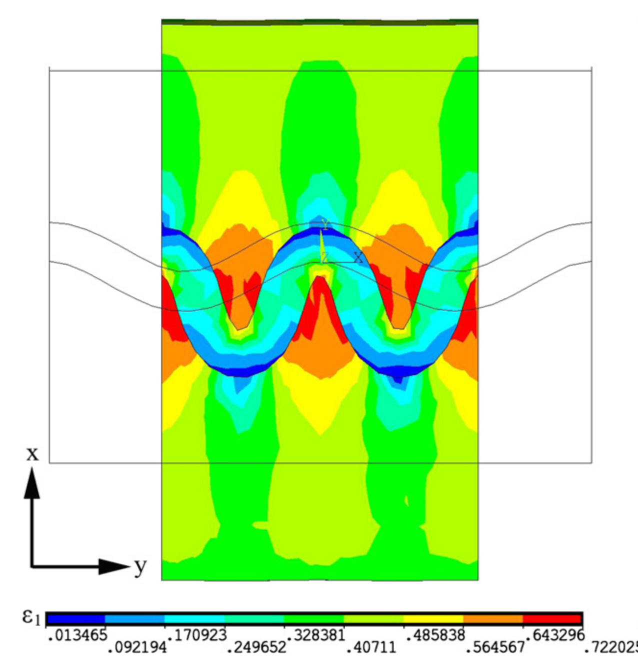

Fig. 7. Finite element models showing deformation of a folded stiff layer in a relatively softer matrix. Details of model are cited in Table 1. Colour contours show the finite strain distributions in both the layer and the matrix.

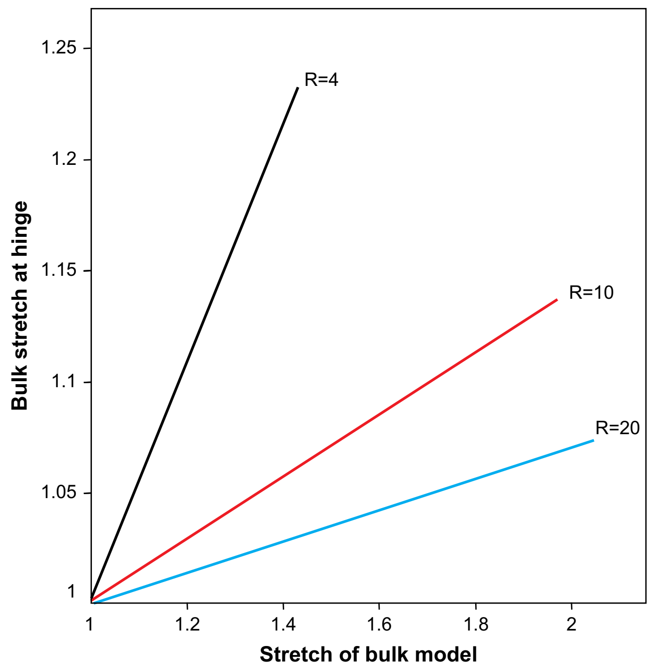

Fig. 8. Variations of quadratic elongation (λ_{1h}) of the fold hinge with the bulk quadratic elongation (λ_{1M}). R: viscosity ratios.

by varying the geometrical irregularity factor ($\zeta_o = 1$ and 3), and $R = 8$ (Fig. 11). The hinge lines of synforms and antiforms are chosen to have the sense of initial curvature in the same direction. However, the sense of curvature of natural sheaths can be in the opposite directions and their magnitudes can be unequal (Alsop and Carreras, 2007). Models with $\zeta_o = 3$ produced almost cylindrical folds (Fig. 11a). In contrast, in models with $\zeta_o = 1$, the hinge curvatures accentuated perceptibly (Fig. 11b). With increasing finite strain, the folds progressively tighten, and increase their non-cylindricity. The degree of non-cylindricity appears to be stronger compared to those simulated based on the strain-partitioning factor alone (Figs. 9c and 11b). It thus appears that active bending of the fold structure may also aid in accentuation of the hinge curvature. However, the fold structures, overall, did not attain the geometry of typical sheath folds. The hinge angle and curvature remain in the order of 110° and 4.9 when the folds are extremely tight.

4. Discussions

4.1. Implications of initial geometrical irregularities of hinge lines

One of the necessary conditions for the development of sheath structures is that there must be initial non-cylindricity or hinge-line irregularities on the fold, irrespective of whether the bulk strain is pure and/or simple shear deformation (Ramsay, 1980; Ez, 2000). A fold initially having a cylindrical curvature cannot produce a non-cylindrical structure in homogeneous deformations even when the magnitude of finite strain is very large. According to our analysis, sheath folds are mostly likely to develop in pure shear if the geometrical factor $\zeta_o < 1$. We thus need to address how folds with such an initial curvatures can form. Ramsay (1980) has argued that layers contain primary undulations prior to deformation, which amplify in developing a fold with an initial non-cylindricity. In such cases the geometrical irregularity of hinge line will depend on the primary features, and its magnitude can be of any amount. However, experimental results presented in Section 1 indicate that the magnitude of this hinge-line irregularity factor for buckle folds in mechanical layers is larger than 1, implying lower hinge-line curvatures. On the other hand, field evidence shows that buckle folds can develop sheath structures (Ghosh and Sengupta, 1984; Alsop and Holdsworth, 2006). FE model results indicate that sheath folds of buckling origin are likely to occur in pure shear only when the initial non-cylindricity is quite large (Fig. 9b). It thus appears that additional factors must play a role in nucleation of folds with higher initial non-cylindricity. Layered or foliated rocks may inherently contain geometrical or mechanical heterogeneities. Buckle instabilities developed in the presence of such heterogeneities can produce folds with large initial hinge curvatures (cf. Marques and Cobbold, 1995; Mandal et al., 2004; Alsop and Holdsworth, 2004). However, we propose this intuitively, and it needs to be tested using further experimental studies.

4.2. Consideration of initial non-cylindrical shape

In order to analyze the passive accentuation of the initial non-cylindrical geometries, different workers have considered the

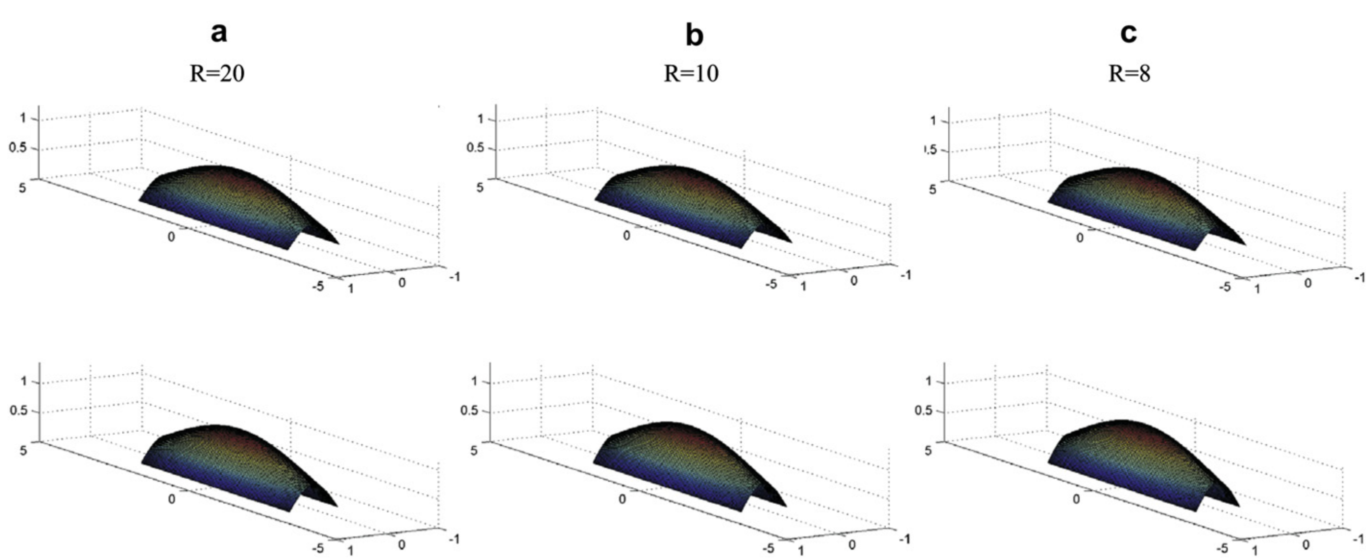

Fig. 9. Simulations of non-cylindrical folds in stiff layers with viscosity ratios $R = 20$, 10 and 8 in (a), (b) and (c), respectively.

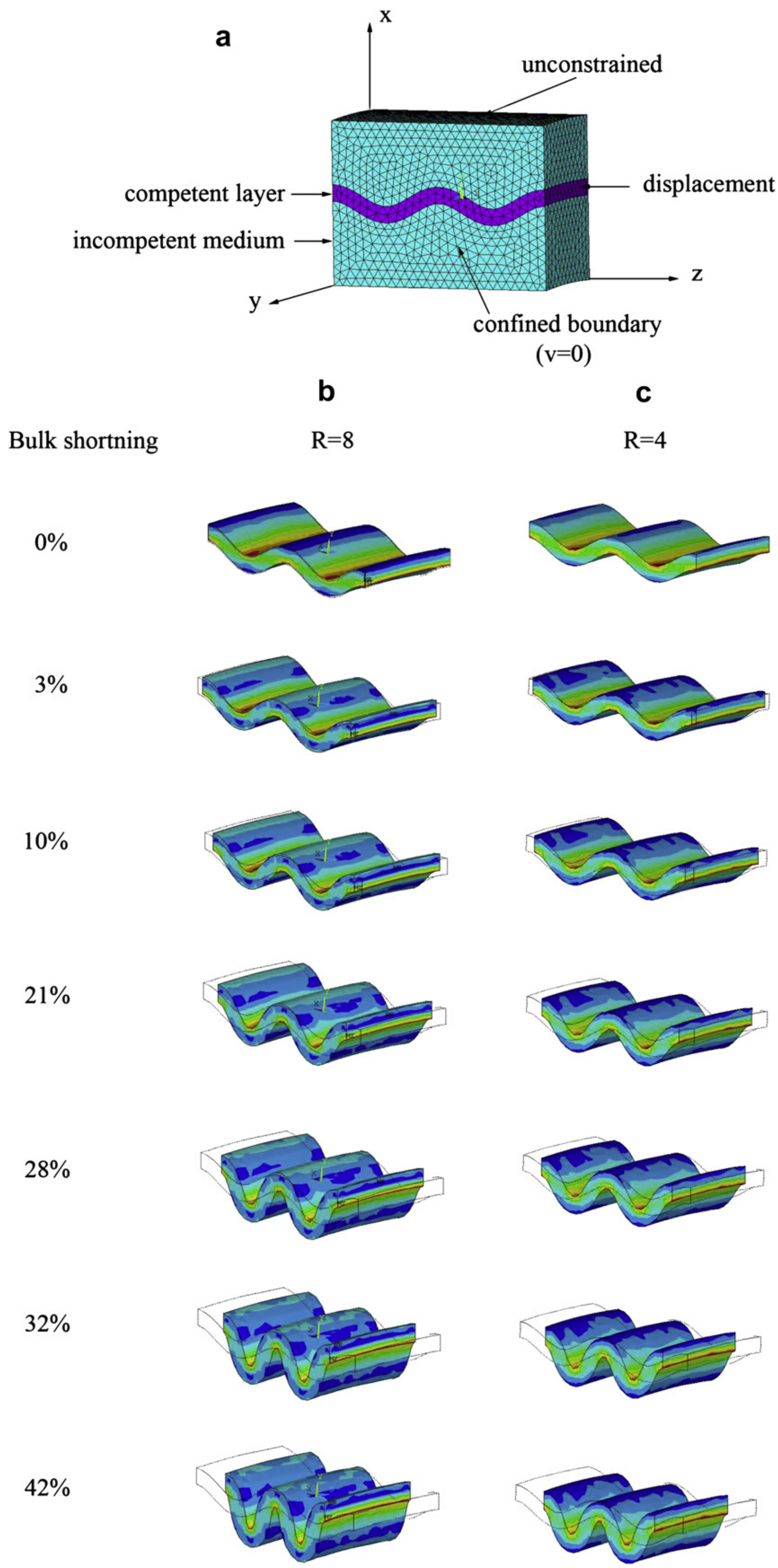

Fig. 10. (a) Three-dimensional finite element model containing a folded stiff layer within an incompetent matrix. (b) and (c) Progressive tightening of non-cylindrical folds with $\zeta_o = 3$ and viscosity ratios $R = 8$ and 4. The initial fold waves are shown in dashed lines. Lower and upper blocks have been removed to show the three-dimensional geometry of deformed fold waves in the stiff layers.

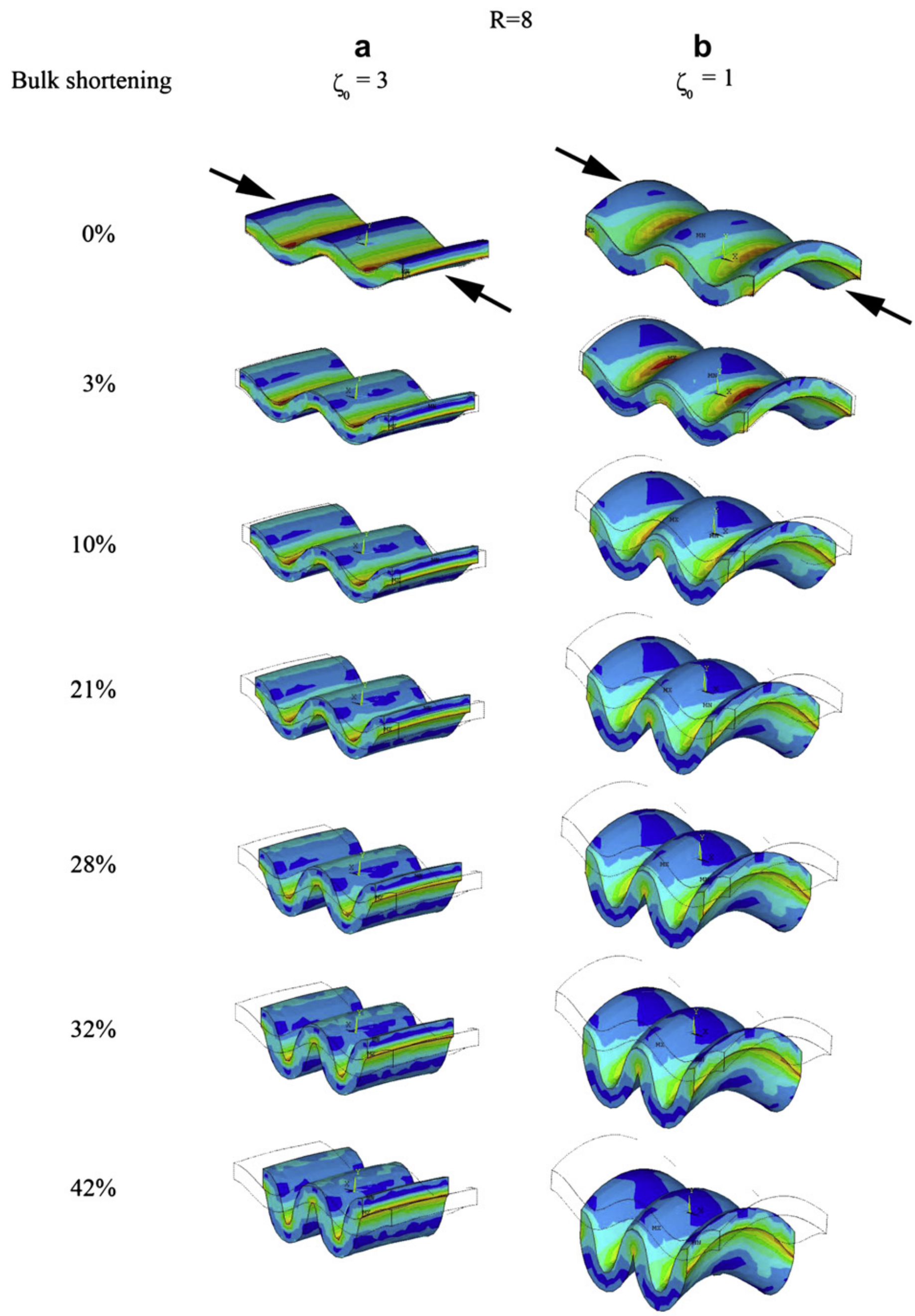

Fig. 11. Development of non-cylindrical folds in competent layers with $\zeta_o = 3$ (a) and 1 (b).

initial shape of rudimentary folds in different ways. Ramsay (1980) assumed that initial non-cylindrical geometry had a distinct line of principal curvature (i.e. the hinge line) and analyzed fold shape modification during later deformations. This approach has been considered in later studies (e.g. Ghosh and Sengupta, 1984; Skjernaa, 1989; Alsop and Carreras, 2007). On the other hand, several workers have considered non-cylindrical deflection with a dome and basin like shape and studied their shape modification and increasing non-cylindricity (e.g. Ez, 2000; Marques et al., 2008). In our analysis we have considered the initial non-cylindrical fold with a hinge line (cf. Ramsay, 1980; Ghosh and Sengupta, 1984). This consideration is merely to deal with a general non-cylindrical curvature of the initial fold, as observed in buckling experiments on layered models where folds develop in the form of wave instabilities (e.g. Ghosh and Ramberg, 1968). Under layer-parallel shortening, folds following their nucleation immediately attain an elongate shape before significant tightening, and develop a hinge line. However, our model can also be applied to folds with both dome like curvature and general non-cylindrical curvature with a hinge line, as it is concerned with the wavy trace of folds obtained on the section perpendicular to the principal shortening direction. This analysis presents an estimate for the initial amplitude of such wavy irregularities that can produce sheath-like geometries following ductile strain in the folded passive or mechanical layers undergoing pure shear deformations.

4.3. Sheath folds in shear zones

The analyses presented in the preceding sections are based on pure shear kinematics. However, sheath folds are quite common in shear zones where the deformations are non-coaxial. In this section we discuss the implications of our model results in view of sheath structures observed in shear zones. Non-coaxial deformations in shear zones are characterized by a combination of strain and rotation tensors. The strain tensor is responsible for shape changes of deformable bodies, whereas the rotation tensor induces an overall rigid rotation. The necessary condition for large shape modifications is a large amount of shortening across the fold axial plane in response to the strain tensor in shear zones. The development of sheath folds is possible also in pure shear, as suggested by Ez (2000), when the magnitude of strain tenors is large. The major difference between sheath fold developments in simple and pure shear follows that the axial planes of folds continuously rotate and tend to orient along the shear plane while accentuating its non-cylindricity in simple shear, whereas those in pure shear accentuate the non-cylindricity, keeping axial planes always parallel to the principal plane of finite strain. However, both types of deformation involve accentuation of initial non-cylindricity in response to the strain tensor components of deformation.

In this study we have used pure shear deformation models merely to show the magnitude of initial non-cylindricity required for the development of sheath folds. This analysis assumes that the accentuation of hinge curvatures is taking place in response to strain tensors. Such accentuations are possible also in simple shear types of deformation, as discussed above. According to our analysis, a sheath-like geometry can develop by accentuation only when the initial hinge curvature exceeds a critical value (i.e. $\zeta_o < 1$). Many workers have shown that sheath folds of buckling origin can from in shear zones. At the initial stage of simple shear deformations, they grow symmetrically, as in pure shear deformation with layer-parallel principal shortening (Ghosh, 1993). Our experimental findings indicate that buckle folds are likely to nucleate with initial hinge curvatures much smaller than the critical value required for sheath development (Fig. 3). Based on our field observations from the Singhbhum Shear zones, we propose a *hinge propagation model* to explain formation of folds with larger initial non-cylindricity.

The Singhbhum Shear zones in India (Ghosh and Sengupta, 1987; Mukhopadhyay, 2001; Mishra, 2006) record gentle to extremely tight or isoclinals folds. Geometrical features, such as wavelength vs layer thickness relationships and, the presence of multi-order fold waves, indicate that folds have developed by the process of buckling. The folds can be observed in three dimensions. Gentle folds occur discretely in several locations with a three-dimensional curvature (Fig. 12a). The folds are diversely oriented with respect to the lineation that is oriented parallel to the shear direction. The hinge lines of gentle, elongate folds are inclined to the lineation at angles varying from 90° to almost zero. In places, the fold hinges are at low angles to the shear direction (Fig. 12b). Nucleation of buckle folds in shear zones is controlled by discrete weak mechanical heterogeneities parallel to the mylonitic foliation (Alsop and Holdsworth, 2002; Mandal et al., 2004). Thus, their diverse orientations might have resulted from varying orientations of inherent weak zones in the foliated rocks. We propose that randomly distributed discrete folds coalesce with one another, giving rise to curved hinge lines (Fig. 13). Gentle folds with large non-cylindricity were observed in the Singhbhum Shear zone (Fig. 12c), which appears to have developed by the process of hinge coalescence of adjoining discrete folds (cf. Dubey and Cobbold, 1977). The folds can accentuate their hinge-line curvature further by homogeneous strain, and attain a strongly non-cylindrical geometry (Figs. 12d and 13). Based on our observations from the field and experiments, our model predicts that sheath folds are likely to form in shear zones only when they achieve large non-cylindricity ($\zeta_o < 1$) by hinge coalescence at the initial stage.

This hinge coalescence model is proposed as one of the possible mechanisms. Some other mechanisms can also give rise to large initial non-cylindricity. For example, ductile flow may be perturbed locally due to heterogeneities in the mechanical properties of the foliated rocks leading to differential layer-normal shearing and 3D strain. Buckling processes in such a condition can produce folds with strong spatial curvatures, and thereby give rise to a significant non-cylindricity in the rudimentary fold structures (Alsop and Holdsworth, 2002; see also Holdsworth and Pinheiro, 2000; Holdsworth et al., 2002). We do not rule out such a mechanism of flow heterogeneity in controlling development of high initial non-cylindricity of folds in natural ductile shear zones.

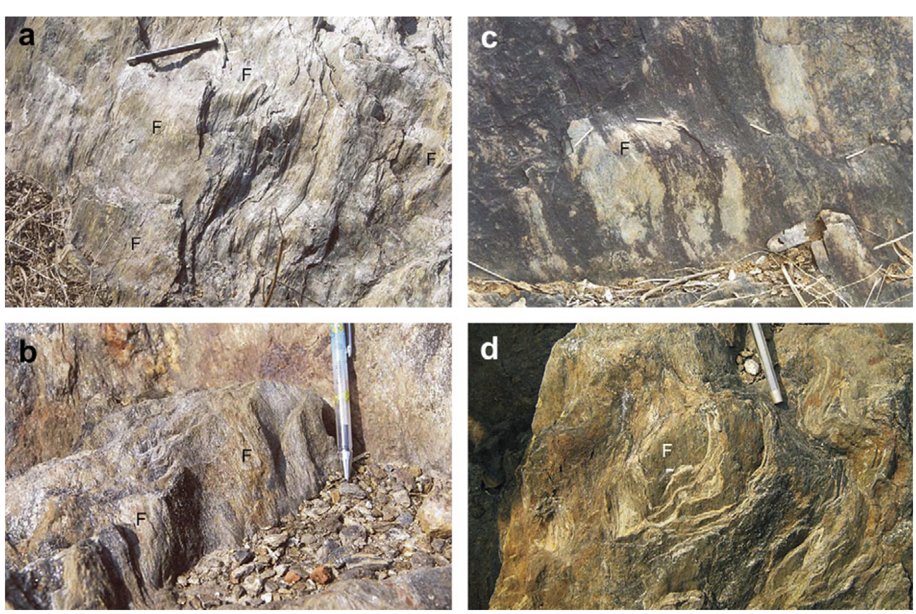

Fig. 12. Plane non-cylindrical folds in the Singhbhum Shear zone, eastern India. (a) Nucleation of discrete folds on mylonitic foliations. (b) Gentle fold waves with hinge lines at low angles to the mineral lineation. (c) Coalescence of two adjoining folds, giving rise to sinuous hinge lines (shown by match sticks). (d) Fold with sheath-like hinge curvature (see near the pen tip).

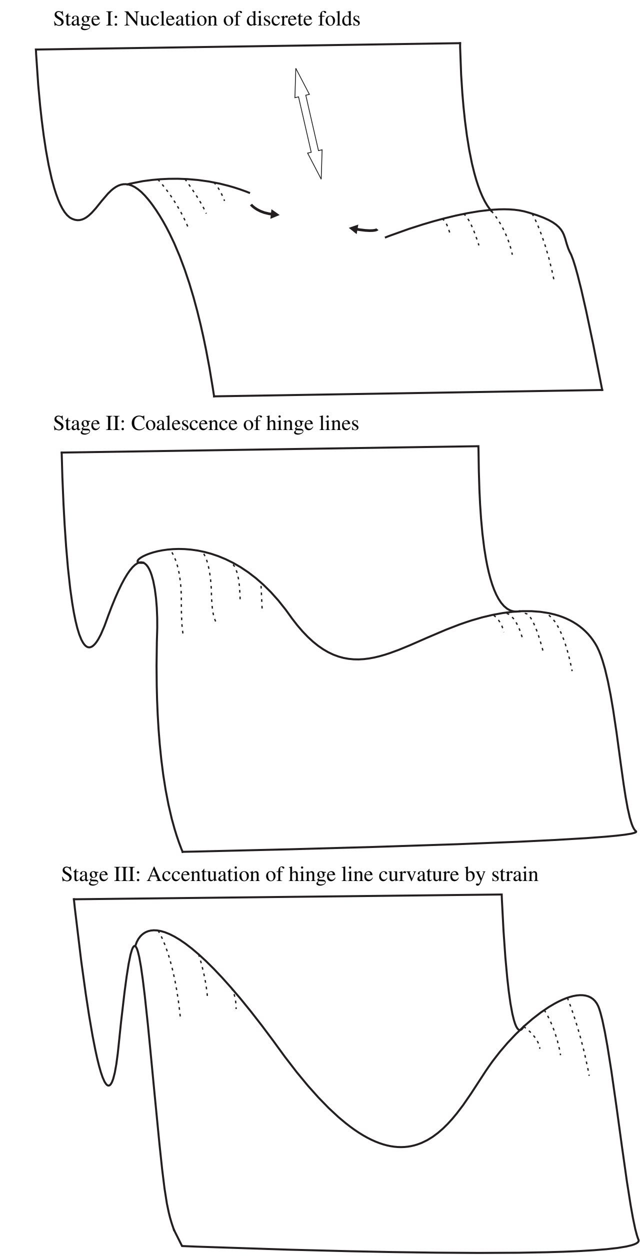

Fig. 13. Hinge coalescence model for formation of folds with strongly curved hinge lines. Discrete folds nucleate randomly, and propagate laterally. Coalescence of spatially side-stepped folds produces curved hinge lines, as observed in the Singhbhum Shear zone (Fig. 12c).

4.4. Limitations

1) Our analysis assumes that the axial planes always coincide with the principal plane of strain. However, there can be an angular relationship between them, as reflected by the development of transecting cleavages in natural sheath folds (cf. Holdsworth et al., 2002; Alsop and Holdsworth, 2004). Secondly, the axial surface of non-cylindrical folds may be somewhat non-planar. Thus, the present results are applicable to idealistic planar fold systems.
2) We ran numerical experiments on passive accentuations, considering tight (interlimb angle 60°) initial folds. However, initial folds can be of primary (penecontemporaneous deformations) and secondary (buckling or heterogeneous flow) origin, and there can be wide variations in their tightness at the time of passive accentuation. The study thus needs to be advanced to test the influence of initial interlimb angle on the evolution of sheath folds.
3) Our simulation experiments were run with antiforms and synforms with hinge-line curvatures in the same direction. However, the sense of curvatures can be in the opposite directions, and their accentuation can take place at different magnitudes (Alsop and Carreras, 2007).

5. Conclusions

The principal outcomes of this study are:

1) For the development of strong non-cylindricity comparable to that of sheath folds in shear zones, fold hinges must contain an initial geometrical irregularity with amplitudes exceeding a critical value. Folds growing in passive layers can achieve sheath-like geometry only when the amplitude to wavelength ratio is larger than 10^{-1} (i.e. $\zeta_o < 1$).
2) In flattening type strain fields ($1 > k$ value > 0), passive folds with $\zeta_o \geq 1$ are unlikely to produce a sheath-like geometry by accentuation of hinge-line curvatures. This is possible only when $\zeta_o \ll 1$.
3) In case of layers with competence contrast, strain partitioning between fold hinges and the bulk medium is an additional factor, and strongly non-cylindrical structures cannot form when the viscosity ratio is less than 10 and the amplitude/wavelength ratio of hinge irregularity is in the order of 10^{-1}.
4) Initially non-cylindrical folds in a high-viscosity layer can undergo flexural bending of the hinge zone, and their non-cylindricity increases by a combined effect of strain-induced curvature accentuation and bending processes. However, the folds remain virtually cylindrical when the irregularity amplitudes are less than 10^{-1}, (i.e. $\zeta_o > 1$) irrespective of viscosity contrast. A typical plane non-cylindrical structure is produced for large amplitudes. Sheath-like structures do not develop even when the viscosity ratio is low, e.g. 4.

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